spin-1 version of Putnam argument

Consider a state Ψ for a spin-1 system defined by
$$\Psi = \frac{1}{\sqrt{2}}(\Psi_1 + \phi_3) \qquad (1)$$
where $\phi_1\ \phi_2\ \phi_3$ is (S_z)-basis $\{ |S_z=-1\rangle, |S_z=0\rangle, |S_z=+1\rangle \}$.

and
$$\Psi_1 = c_1\phi_1 + c_2\phi_2$$
so Ψ_1 is orthogonal to ϕ_3

and we can write (1) in form
$$\Psi = \frac{1}{\sqrt{2}}(c_1\phi_1 + c_2\phi_2 + \phi_3) \qquad (2)$$
Now measure S_z and find result $S_z = -1$

then from (2) we know the value of P_{ϕ_1} (viz 1)

But, according to Putnam

in virtue of looking at form (1) for Ψ

we also know the value of P_{Ψ} the incompatible projector P_{Ψ_1} (viz 1),

whereas my claim is that the weaker claim that we know is that the compatible projector $P_{\phi_1} + P_{\phi_2}$ has the value)

(note that $P_{\Psi_1} < P_{\phi_1} + \phi_2$)

so $P_{\Psi_1} \Rightarrow P_{\phi_1} + P_{\phi_2}$ and in particular

$P_{\phi_1} \Rightarrow P_{\phi_1} + P_{\phi_2}$. But $P_{\phi_1} + P_{\phi_2} \not\Rightarrow P_{\Psi_1}$)

Logic, Quanta

~~Putnam~~ and the Two-Slit Experiment

Michael Redhead
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The two-slit experiment has always provided a major focus for debates on the interpretation of quantum mechanics (QM). In brief the experiment consists in allowing a beam of electrons of well-defined momentum to impinge on a screen which incorporates two parallel slits, and detecting the electrons emerging from the slits on a second screen, equipped for example with a photographic emulsion which will respond to the impact of an electron. The intensity of the beam can be reduced so that on average at a given time only one electron is passing through the ~~screen with~~ device comprising the two screens the one with the two slits and the detector screen. But nevertheless QM predicts that an 'interference' pattern will build up on the detector screen, quite unlike the mere summation of the patterns one would obtain if one or other of the slits were open but not both together. The electrons 'behave' like localized corpuscles in respect of their detection at the second screen but in respect of the interference pattern the 'behaviour' is characteristic of classical wave behaviour. So in this experiment it appears that the 'electrons' are displaying

wave-like behaviour in their passage through the slits in the first screen, but particle-like behaviour in respect of their detection at the second screen. But how can an electron behave both like a wave and like a particle? That is the essence of the paradox posed by the two-slit experiment.

new para

Referring to Fig. 1. S_1 and S_2 are the two slits in the first screen, and D_j is a small volume element enclosing a piece of emulsion that can record if the electron hits ~~the hits~~ the detector screen at a point located in D_j.

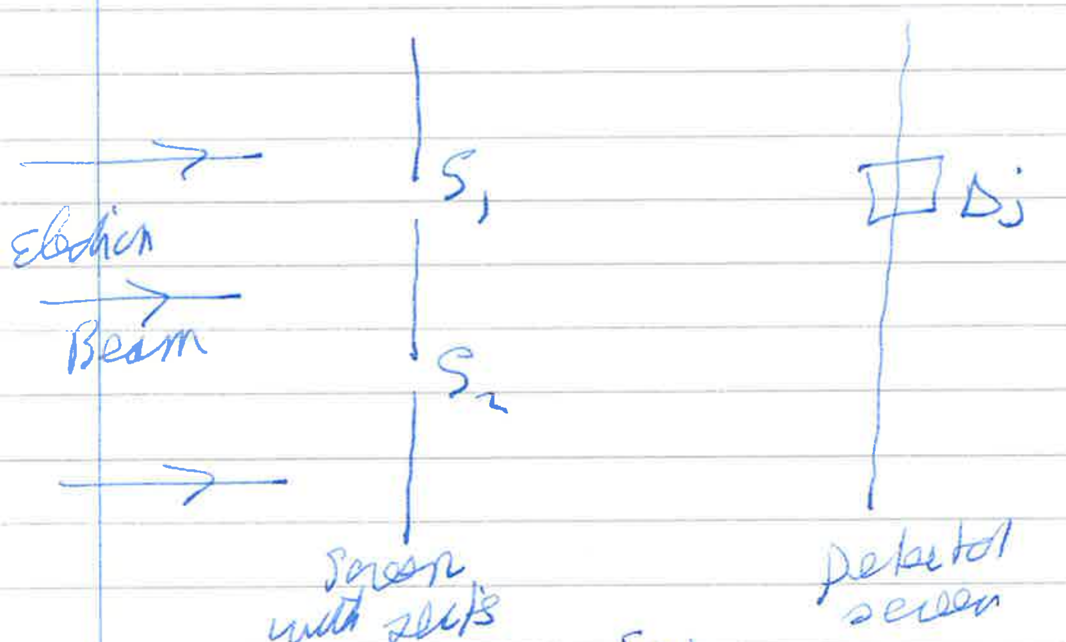

Fig. 1.

Denote by A_1 the proposition asserted at the time t that the electron is detected that the electron had passed through the slit A_1 ~~and~~ at the earlier time t' at which the beam impinges on the first screen and by A_2 the proposition at t, that the electron had passed through A_2 at t'. Furthermore denote by R_j the proposition that the electron later triggers the detector at time t in cell D_j.

then we are concerned in explaining the two-slit experiment with evaluating ~~Prob (R_j) +~~ the conditional probability Prob (R_j | $A_1 \vee A_2$).

very poor

Let us first see how, treating the electron as ~~particles~~ a particle 'which has passed through one or other slit,' leads to the wrong sort of pattern on the detector screen.

Let us evaluate Prob($R_j | A_1 \vee A_2$) according to classical ideas on the meaning of conditional probability

$$\text{Prob}(R_j | A_1 \vee A_2) = \frac{\text{Prob}(R_j \wedge (A_1 \vee A_2))}{\text{Prob}(A_1 \vee A_2)} \qquad (1)$$

$$\cancel{= \text{Prob}(R_j \wedge (A}$$

$$= \frac{\text{Prob}\left((R_j \wedge A_1) \vee (R_j \wedge A_2)\right)}{\text{Prob}(A_1 \vee A_2)}$$

where we have used the distributive law of classical logic to write

$$R_j \wedge (A_1 \vee A_2) = (R_j \wedge A_1) \vee (R_j \wedge A_2) \qquad (2)$$

~~But since $A_1 \wedge A_2$ is contrad. a logical contradiction we have.~~

Now

~~Prob ($R_j | A_1 \vee A_2$) = Prob ($A_j \wedge A_1$)~~

$$\text{Prob}\left((R_j \wedge A_1) \vee (R_j \wedge A_2)\right)$$

$$= \text{Prob}(R_j \wedge A_1) + \text{Prob}(R_j \wedge A_2) - \text{Prob}(R_j \wedge A_1 \wedge A_2)$$

$$= \text{Prob}(R_j \wedge A_1) + \text{Prob}(R_j \wedge A_2)$$

$$\cancel{= \text{Prob}(R_j|A_1) \cdot \text{Prob}(A_1) + \text{Prob}(R_j|A_2) \cdot \text{Prob}(A_2)}$$

since $A_1 \wedge A_2$ is a logical contradiction

$$= \text{Prob}(R_j|A_1) \cdot \text{Prob}(A_1) + \text{Prob}(R_j|A_2) \cdot \text{Prob}(A_2)$$

So, finally

$$\text{Prob}(R_j \mid A_1 \vee A_2)$$

$$= \cancel{\text{Prob}(R_j \mid A_1) \times \frac{\text{Prob}(A_1)}{\text{Prob}(A_1 \vee A)}}$$

$$= \text{Prob}(R_j \mid A_1) \times \frac{\text{Prob}(A_1)}{\text{Prob}(A_1) + \text{Prob}(A_2)}$$

$$+ \text{Prob}(R_j \mid A_2) \times \frac{\text{Prob}(A_2)}{\text{Prob}(A_1) + \text{Prob}(A_2)}$$

$$= \tfrac{1}{2}\,\text{Prob}(R_j \mid A_1) + \tfrac{1}{2}\,\text{Prob}(R_j \mid A_2) \quad \text{--- (3)}$$

if we assume a uniform incident beam, so $\text{Prob}(A_1) = \text{Prob}(A_2)$.

new para

Now (3) is just the equally weighted summation of the patterns we would get if S_1 or S_2 were alone opened and exhibits none of the features of QM interference.
In order to reproduce such interference it would be necessary to assume that $\text{Prob}(R_j \mid A_1)$ for example depended on whether S_2 was opened or closed but this would admit a mysterious nonlocal action between opening and closing S_2 and what the electron was doing as it went through S_1. It is the hope of avoiding what Reichenbach in his (1944) refers to as 'causal anomalies' that has inspired much of the discussion of the interpretation of the two-slit experiment.

new para

The approach of the ~~Copen~~ orthodox Copenhagen interpretation to this problem is simply to

claim that when the electron is passing through the slits in the first screen, and displaying wave-behaviour, it is meaningless to introduce ~~probab~~ propositions like $A_1 \vee A_2$ which express a typically particle ~~W~~ notion, that the electron goes through one or other slit. So the whole of the above discussion is quite illegitimate according to the Copenhagenists because it involves conditioning on a meaningless proposition. The main argument in Reichenbach's (1944) is to suggest that one can regard $A_1 \vee A_2$ as meaningful in the context of the two-slit experiment but in order to avoid causal anomalies we must regard $A_1 \vee A_2$ as neither true nor false, but accorded a third truth value viz. indeterminate. This approach of employing a three-valued logic to interpret the two-slit experiment was taken up enthusiastically by Putnam in his (1957). But in his (1965a) he was quite dismissive: "In Reichenbach's approach ... it is simply assumed that statements about macro-observables have the conventional two truth values while statements about micro-observables may have a third truth value; but this radical dichotomy between macro- and micro-observables is not derived from anything, but simply built into the theory ad hoc". In brief, the three-valued logic approach formalizes but does not explain the phenomenon of interference. In his (1965a) Putnam entitled 'A Philosopher looks at Quantum Mechanics'

Putnam makes no reference to quantum logic and concludes "NO satisfactory interpretation of quantum mechanics exists today"

new para

But then in his (~~1968~~ 1969) Putnam took up the idea of a bivalent but non-distributive logic as a resource for removing the paradoxes associated with the interpretation of QM. Technically the idea goes back to Birkhoff and von Neumann, who showed in 1936 that a in a certain 'sense', a non-distributive logic could be "read off" the mathematics of the Hilbert space formulation of quantum mechanics. To see what is going on we present a brief résumé of the non-distributive logic approach concentrating on those features emphasized by Putnam in his (~~1968~~ 1969) and his (1974).

In classical physics the state of a system is identified with a location in phase-space and we can introduce elementary propositions P of the form ~~e(U)~~ e(P) specifying that the state of the system lies in the subset P of the phase-space Ω. The logical connectives, conjunction disjunction and negation acting on the elementary propositions now translate into the familiar Boolean operations of intersection, union, and set-theoretic complement under the correspondence associating propositions P with subsets P. In QM the ~~state of a system is identified with~~ maximally specified state of a system is identified with a ray or one-dimensional subspace of a Hilbert space $\mathcal{H}$. We can now introduce elementary propositions u ~~which~~ of the form e(U) specifying that

the state of the system lies somewhere in the subspace U, of H, and the logical connectives '$\wedge$' '$\vee$' and '$\neg$'' are now defined by their translation into the lattice ~~operators~~ operations of meet, join and ortho complement in the lattice of subspaces of H. The resulting 'logic' is easily seen to be non-distributive. But now comes the decisive step. We introduce new elementary propositions of the form $(\Delta)_Q$ which assert that the the observable Q possesses a value which lies in the Borel subset Δ of possible values for Q. We now introduce the idea of a 'real' state or Putnam state as it is described in Redhead (1987) which is to be sharply distinguished from the QM state. ~~The Putnam state is defined as the QM state~~ The proposition $(\Delta)_Q$ is identified with the proposition $e'(U)$ specifying that the Putnam state lies in the subspace U of H which is now defined by the statement that if the QM state were confined to U then with probability one the result of measuring Q would lie in Δ. It is easily checked that $U = \mathrm{ran}\, P^Q(\Delta)$ where $P^Q(\Delta)$ is the projection operator associated with the Borel set Δ via the projection-valued measure associated with the hypermaximal operator that represents Q in the Hilbert space formulation of QM. The logical connectives acting on the $(\Delta)_Q$ propositions are now to be understood as translating into the lattice operations on the subspaces U via the correspondence

$$(\Delta)_Q \mapsto e'(U).$$

The whole scheme is really quite complicated

and we have tried to summarize the situation in a schematic form in Fig. 2.

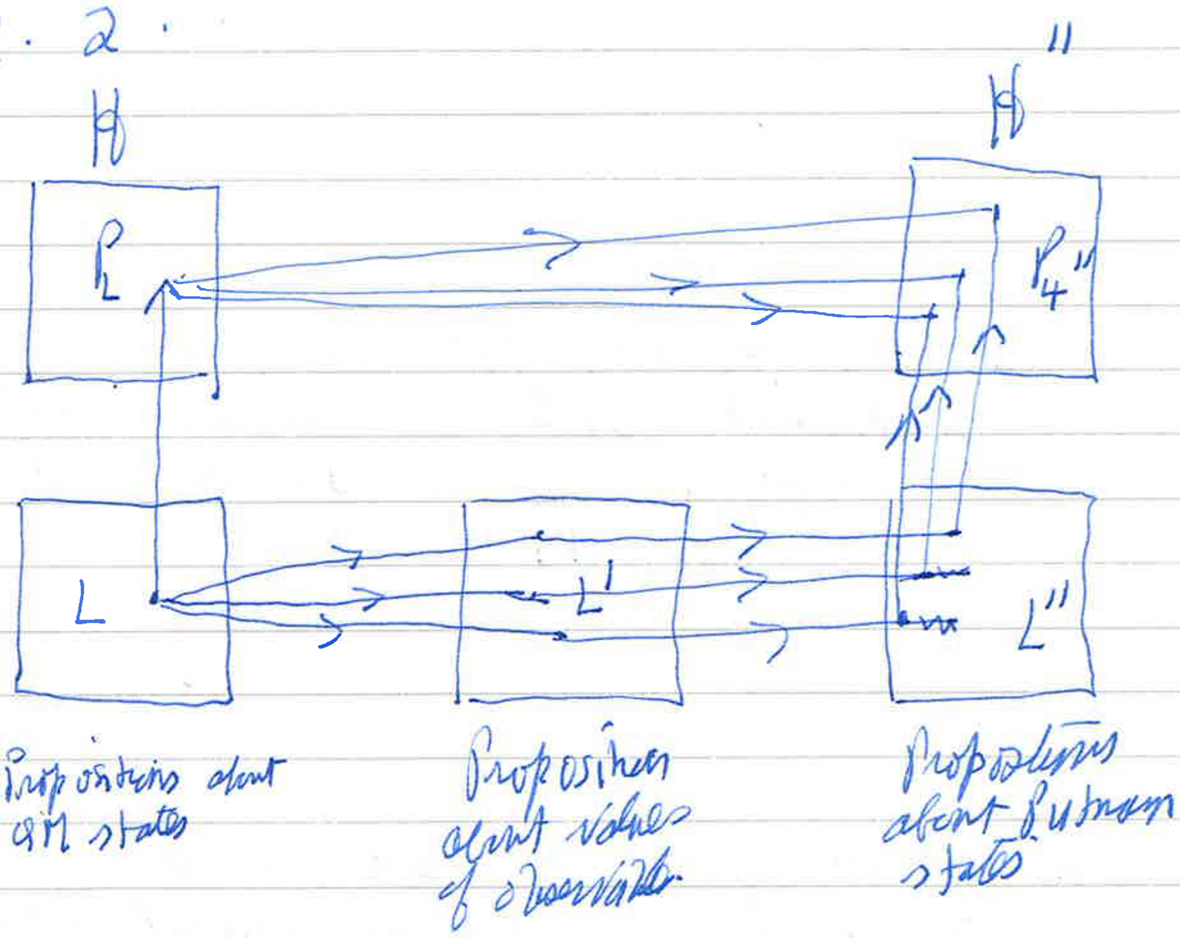

Fig. 2

L denotes a proposition about the specification of the QM state of the form $e(v)$ as described above.
L' is a proposition of the form $(\Delta)_Q$ telling us about the values of observables. ~~Each~~ On a realist construal of QM each L-proposition is associated with a non-denumerable infinity of L'-propositions. Each L'-proposition is now associated via the correspondence $(\Delta)_Q \to e'(v)$ with an L''-proposition specifying the location of what we have called the Putnam state.
Each L-proposition is associated with a subspace of the Hilbert space $\mathcal{H}$, and may be identified via its range with a unique projection operator P_L acting on $\mathcal{H}$. Similarly each L''-proposition is associated with a

Insertion

A_1 ensures that the quantum-mechanical state is associated with the ray N, whose existence is asserted in A_1.

Insertion

Let us restrict discussion for the moment to a finite-dimensional Hilbert space, so avoiding problems of observables with a continuous spectrum, then

unique projection operator $P_{L''}$ acting on a Hilbert space H'' the space of Pythagorean states, which is formally identical with H. But conceptually H'' and H must be sharply distinguished, a one-many map existing between the P_L and the $P_{L''}$ as ~~sh~~ illustrated in Fig. 2.

new para

In order to complete the logical scheme we require the specification of a truth valuation mapping propositions such as L or L'' onto the two-element Boolean algebra of truth values, 0 and 1. For L-propositions, denoting $e(U)$ by u, it appears that ~~we~~ we require the following admissibility criterion

Bold

display

A_1: $Val: \{u\} \to B_2$ is an admissible valuation iff there is a one-dimensional subspace N such that for every subspace M, $Val(m) = 1$ iff $N \leq M$

equation #

where $\leq$ denotes the subspace relation.

For the L''-propositions the situation is more subtle. extension For a proper realist semantics we would like to enforce the admissibility criterion

Bold

display

A_2: $Val: \{u\} \to B_2$ is an admissible valuation iff the following conditions are satisfied

1. $Val(u) = 1$ iff $Val(\neg u) = 0$
2. if $Val(n) = 1$, and $N \leq M$, then $Val(m) = 1$
3. In any orthonormal basis $\{|q_i\rangle\}$ of H'', where $\{q_i\}$ are the eigenvalues of some maximal observable Q, $Val(q_j) = 1$ for some j and hence from (1) and (2) $Val(q_i) = 0$, $\forall i \neq j$.

Here q_j is a convenient shorthand for the

Insertion

II § It will also follow that nonmaximal observables are assigned unique definite values as a result of $\mathbf{A_2}$. (See Redhead (1987) p. 165.)

θ Effectively Putnam is claiming.

projection $e'(Q_j)$ where Q_j is the one-dimensional subspace generated by the eigenvector $|q_j\rangle$.

new para

The third condition is the crucial one ensuring that every maximal observable has a unique definite value, thus implementing our realist intentions # [illegible]

The trouble is that there simply are no A_2-admissible valuations for Hilbert spaces of dimension ≥ 3.

new para

Faced with this situation Putnam proceeds as follows. Consider a maximal observable Q in a Hilbert space of dimension $N \geq 3$. Then Putnam identifies the statement Q has a value with the disjunction $q_1 \vee q_2 \vee \cdots \vee q_N$. This is not only true but tautologically so. However, says Putnam, this does not mean there is some specifiable j for which q_j is true. Now in classical logic the disjunction carries an existential commitment. We can write $(\exists i)\, q_i \equiv q_1 \vee q_2 \cdots \vee q_N$. Putnam is effectively retaining this ~~equi~~ classical result but using it to define ~~even~~ what he means by $(\exists i)\, q_i$. ~~As Putnam puts it:~~

that

Q has a value but there is no value which it has! Consider now some other maximal observable R with eigenvectors $\{|r_i\rangle\}$ distinct from $\{|q_i\rangle\}$. So the associated operators $\hat{Q}$ and $\hat{R}$ do not commute (Q and R are so-called incompatible observables).

C

Then $(\exists j)\, r_j \equiv r_1 \vee r_2 \cdots \vee r_N$ is also tautologically true, ~~so~~ Indeed. Consider the two statements

Display

$$\begin{cases} S_1 : (\exists i)\, q_i \wedge (\exists j)\, r_j \\ S_2 : (\exists i)(\exists j)\, (q_i \wedge r_j) \end{cases}$$

S_1, according to Putnam, expresses realism

and is tautologically true. In classical logic S_1 and S_2 are logically equivalent, but not so in quantum logic. S_2 indeed is a logical contradiction, and this is regarded by Putnam as expressing complementarity, that although Q and R individually have values, it is contradictory to assert that they possess simultaneous values. It is, of course the failure of the distributive laws that so allows us to deny that S_1 and S_2 are equivalent propositions and leads to reconcile realism and complementarity.

new para

In his (1969) Putnam tried to apply these ideas to elucidating the two-slit experiment. He pointed out that the derivation of the empirically incorrect summation result (3) depended on employing the distributive law (2). If (2) was disallowed we would be prevented from getting the wrong result — of course that is a rather limited objective, quite insufficient ~~from~~ for showing how to get the right result! But it ~~is~~ was pointed out almost immediately by Gardner in his (1971) that in the particular case in question the distributive law is in a certain trivial sense, actually true. To see what is going on let us identify the propositions A_1, A_2 and R_j with the relevant projection operators. For A_1 this is $P_{|\psi_1\rangle}$ the projector onto the state $|\psi_1\rangle$ which is the time-evolution at time t of the state which would arise at time t' if the slit A_1 alone were open. Similarly A_2 is associated with the projector $P_{|\psi_2\rangle}$ where $|\psi_2\rangle$ is the time-evolution at time t of the state which would arise

at time t' if the slit S_2 alone were open. Finally P_i is associated with $P_i = \int_{x \in \Delta_i} dP(x)$, where $P(x)$ generates the projection-valued measure associated with the position operator X.

new para Gardner then pointed out that

$$\left.\begin{aligned} P_i \wedge P_{|\psi_1\rangle} &= 0 \\ P_i \wedge P_{|\psi_2\rangle} &= 0 \\ P_i \wedge (P_{|\psi_1\rangle} \vee P_{|\psi_2\rangle}) &= 0 \end{aligned}\right\} \qquad (4)$$

where 0 denotes the null-projector. So the RHS and LHS of Eq. (2) come out to be trivially equal, each side being the null-projector. so each ~~of the conjunctions in (4) represents a logical contradiction~~. side of the equation represents a logical contradiction!

new para Essentially what ~~Gardner~~ Gardner was pointing out was that states picked out by P_i are never in the linear span of $|\psi_1\rangle$ and $|\psi_2\rangle$. (A mathematically rigorous proof of this result for the two-slit experiment was provided by Gibbins and Pearson in their (1981).

new para Faced with this situation Friedman and Putnam gave a quite different analysis of the two-slit experiment and its connection with quantum logic in their (1978). But this 1978 paper cannot really be understood without taking account of ~~Putnam~~ the general way in which Putnam's views on quantum mechanics changed during the 1970s, in particular his rejection of the

idea that knowing simultaneous values for incompatible observables would be impossible since it would correspond to knowing a logical contradiction. Indeed in 1981 Putnam published a crucial paper entitled 'Quantum Mechanics and the Observer', in which he repudiated his argument ~~to~~ for believing that ~~one could~~ ~~could~~ ~~doubt~~ it was not contradictory to know ~~the values~~ such simultaneous values. We shall analyse the content of this paper ~~in a~~ shortly ~~moment~~. But for the moment let us see how ~~the conclusion~~ this conclusion appears to motivate the 1978 approach. The fact that incompatible propositions are logically contradictory corresponds, as we have seen, to the fact that the meet of the projectors corresponding to the associated Putnam states is the null-projector. This arises because we are using a lattice theoretic (LT) version of quantum logic in which the binary logical connectives are interpreted in terms of the lattice operations which are defined for all pairs of elements in the (projection) lattice. But already in their (1967) Kochen and Specker had employed a partial-Boolean-algebra (PBA) version of quantum logic in which the binary connectives are restricted to compatible pairs of projectors lying that is in a common Boolean sub-algebra of the full non-Boolean projection lattice of ~~the~~ Hilbert space. This PBA version of quantum logic is exactly suited to

Footnotes

1. By transition probability we mean here, not the probability of a transition from one quantum state to another but the change or revision in a probability assignment to a proposition occasioned by a transition from one quantum state to another.

what Putnam has in mind, the quantum-logical contradictions which he had previously identified with the impossibility of simultaneously knowing the values of incompatible observables could no longer be formulated in the PBA version of the logic. But there now arises a problem in formulating a conditional probability in terms of a joint probability as in (1) since the conjunction of the incompatible propositions P_i and $P_1 \vee P_2$ is no longer allowed. So a new formulation of conditional probability has to be invoked in terms of a transition probability" ~~understood.~~ in the following way". We begin with some terminological conventions. We shall use the symbol P for a projector to denote ambiguously (1) the projection operator P (2) the observable P, associated with the projection operator P, (3) the subspace of Hilbert space which is the range of P (4) the proposition: $[P] = 1$, where $[P]$ denotes the value of P, (5) the proposition: The state vector lies in the range of P. Moreover if P is associated with the Borel set Δ and the projection-valued measure associated with an observable Q, then (6) the proposition: $[Q] \in \Delta$.

Note that senses (4) and (6) are equivalent under the assumption of FUNC viz.

Redhead

$$\text{FUNC}: \quad [f(Q)] = f([Q])$$

for any Borel function f of the observable Q.

Question
(The projection postulate)

Question
has made a transition to

The senses (a) or (b) of the symbol P should always be clear from the context. Finally we use the convention that $P|\phi\rangle$, for an arbitrary QM state $|\phi\rangle$, shall always mean the normalized state

$$\frac{P|\phi\rangle}{\|P|\phi\rangle\|}$$

new para

With these conventions in mind we now explain how to understand the conditional probability $\mathrm{Prob}_{|\phi\rangle}(F|P)$ as a transition probability. We understand this quantity as the probability that the proposition F is true given that the initial QM state is $|\phi\rangle$ and that a *maximal* measurement has established the value one for a *one-dimensional* projector P. According to standard ideas in the theory of measurement[*] the QM state after the measurement is $P|\phi\rangle$ and hence

$$\mathrm{Prob}_{|\phi\rangle}(F|P) = \mathrm{Prob}_{|\phi\rangle}(F \,|\, P_{P|\phi\rangle})$$

$$= \mathrm{Prob}_{P|\phi\rangle}(F) \quad \text{--- (5)}$$

But this analysis for maximal measurements cannot be applied as it stands to the two-slit experiment, where we have to evaluate $\mathrm{Prob}_{|\phi\rangle}(R_j \,|\, P_1 + P_2)$, where we have replaced $P_{|\psi_1\rangle}$ by P_1 and $P_{|\psi_2\rangle}$ by P_2 in our earlier notation, but the important point to note here is that $P_1 + P_2$ is a two-dimensional projector (i.e. its range is two-dimensional).

Postscript 2

Quantum

* { In fact what has been derived here is just the so-called Lüders rule for extending the projection postulate to nonmaximal measurements. } For detailed reaction to the Friedman–Putnam proposal and its merits vis-à-vis the Copenhagenist treatment of the two-slit experiment see Hellman (1981), Bub (1982) and Stairs (1982). In this paper we concentrate on a different aspect of the ~~discussion~~ problem, viz. the relation said [illegible] the argument in Putnam (1981)

Quantum

Lewis Lewis

① Substitutivity of materially equivalent propositions in conditionalization is of course not generally a truth-preserving move. This is particularly obvious if we understand conditional probabilities as conditionals with a probabilistic consequent so $Prob(A|B) = p$ is analysed as $B \;\Box\!\!\rightarrow (Prob(A) = p)$. Since any two false propositions are materially equivalent, we cannot expect substitution to preserve the truth-value of the counterfactual conditional. But if the material equivalence is provable from some background proposition then, on the Lewis analysis of counterfactuals, for example, substitution is permissible. The question of whether substitution is also allowed when the material equivalence of the antecedents in the counterfactual is only provable from quantum-logically from a background proposition which cannot be conjoined with either of the antecedents may be regarded as problematic.

So (5) cannot be applied directly.

But Friedman and Putnam also note that in quantum logic

$$P_\phi \Rightarrow P_{(P_1+P_2)|\phi\rangle} \leftrightarrow P_1+P_2 \quad (6)$$

where $\leftrightarrow$ denotes material equivalence and $\Rightarrow$ denotes logical ~~implication~~ entailment in the quantum logic. Since $P_{(P_1+P_2)|\phi\rangle}$ and P_1+P_2 are compatible propositions the definition of material equivalence is just the classical one

$$A \leftrightarrow B \equiv (A \supset B) \cap (B \supset A)$$

where $\supset$ is the ~~usual~~ material conditional of classical logic.

In virtue of (6) Friedman and Putnam claim that we can substitute the one-dimensional projector $P_{(P_1+P_2)|\phi\rangle}$ for the two-dimensional projector P_1+P_2 and hence, applying (5) obtain

$$\begin{aligned} &\mathrm{Prob}_{|\phi\rangle}(R_j \mid P_1+P_2) \\ &= \mathrm{Prob}_{|\phi\rangle}(R_j \mid P_{(P_1+P_2)|\phi\rangle}) \\ &= \mathrm{Prob}_{(P_1+P_2)|\phi\rangle}(R_j) \quad (7) \end{aligned}$$

which yields of course the correct empirical interference pattern.

Be that as it may ~~But~~ this solution to the two-slit problem is certainly ingenious and we turn now to consider how the argument connects with Putnam (1981).

In fact we shall show that knowing simultaneous values for incompatible observables cannot be independently established as Putnam seems to think but can only be established on the basis of a quantum-logical argument based on (6). ~~There is nothing viciously circular here indeed a pleasing consistency in the whole treatment arises.~~

new para

Putnam's argument that one can simultaneously know the values of incompatible observables is based on his analysis of a thought experiment which we now explain. The exact details have been somewhat simplified so as to bring out the point at issue in the clearest possible way. We consider a particle which can escape through a shutter easily opened in the wall of a small confining box, and if it does so escape is detected by a spherical photographic emulsion with its centre located at the shutter (Putnam himself treats of a photon escaping from a box with absorbing walls, so the photon either escapes and is detected or does not escape and is presumably absorbed but this just makes the physics more complicated to represent accurately). The set-up is illustrated in Fig. 3.

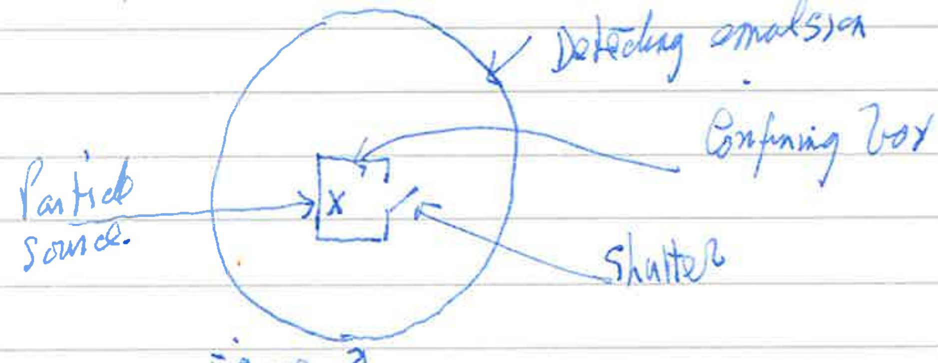

Figure 3.

stet

Gnotus
✗ Later we shall indicate what happens when we describe ~~both the~~ state of the particle and the photographic emulsion, as Pulmann requires.

~~At the time t at which the particle reaches the detector~~ After the shutter has been opened and closed the QM state for the particle can be expressed as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\Psi_{in}\rangle + |\Psi_{out}\rangle\right) \quad (8)$$

where $|\Psi_{in}\rangle$ is a state confined to the interior of the box and $|\Psi_{out}\rangle$ is a state confined to the exterior of the box, and we assume for the sake of simplicity that the shutter is kept open for just such a length of time that the probability of the particle escaping from the box is 1/2.

Putnam himself insists that $|\Psi\rangle$ in (8) is represented in the Hilbert space ~~of the~~ appropriate to the particle and the photographic emulsion. in the present ~~He~~ We prefer to keep simple things simple and let $|\Psi\rangle$ describe just the particle as stated. ~~nothing in what follows is in the slightest way altered by this simplification.~~ * Treating

new para Suppose we divide the whole of space into discrete cells B_i and define projection operators associated with these cells as before. P_i^{in} if B_i lies inside the box and P_i^{out} if B_i lies outside the box.

Then clearly

$$|\Psi_{out}\rangle = \left(\sum_i P_i^{out}\right)|\Psi\rangle \quad (9)$$

and

$$|\Psi_{in}\rangle = \left(\sum_i P_i^{in}\right)|\Psi\rangle \quad (10)$$

where the summation in (9) is over the cells outside the box, and the summation in (10) is over the cells

Exeter
X

At this point it is important to notice that Ψ_{out} should be understood as referring to a superposition of coupled states of the particle and the emulsion, and P_R refer to one component in this entangled superposition in which the mark has appeared in D_R. The above analysis can easily be adapted to the situation, which is the one actually envisaged by Putnam, by considering appropriate projections in the tensor product space of the particle and all the atoms in the emulsion. We leave it as an exercise to the interested reader to check that everything goes through in this more complicated setting in exact parallel with the simple mathematics we have described.

inside the box and we again use our convenient convention that states are only specified modulo normalization.
We also note that

$$\sum_i P_i^{in} + \sum_i P_i^{out} = I \quad (11)$$

and $P_j^{out} \Rightarrow \sum_i P_i^{out}$ (12)

$$\Rightarrow \sum_i P_i^{in} + \sum_i P_i^{out} \quad (13)$$

Since all the P_i^{in} and P_i^{out} are compatible (12) is just the classical result that if the particle is in the j^{th} cell outside the box then it is outside the box, while (13) is the classical result that if the particle is in the i^{th} cell outside the box then it is somewhere (i.e. either inside or outside the box).

somewhere

Now suppose at some appropriate time t we measure $P_{\Delta R}^{out}$, where ΔR encloses a small region of the emulsion and find the value 1 (i.e. a mark on the emulsion appears in ΔR at time t) then we know the proposition $P_{\Delta R}^{out}$ (i.e. at time t the particle was located in ΔR). But Putnam now claims that in virtue of knowing $P_{\Delta R}^{out}$ we also know that the particle is outside the box and ~~know (says Putnam)~~ $P_{\Sigma out}$, ~~which of course does not commute with $P_{\Delta R}^{out}$~~ and this (says Putnam) means knowing the ~~proposition~~ 1 value of $P_{[\Sigma out]}$ which of course does not commute with $P_{\Delta R}^{out}$. * [So the conclusion of the argument is that drawing the

new para

new para

mark on the emulsion in D_R, we thereby know the values of two incompatible observables.

But this just seems confused.

From (12) identifying i with R we have $P_R^{out} \Rightarrow \sum P_i^{out}$, so, in words, if the particle is somewhere specifically outside the box, then it is somewhere outside the box! But P_R^{out} commutes with $\sum P_i^{out}$ so there is no question of knowing incompatible propositions in virtue of knowing that the particle is in D_R and it is somewhere outside the box.

new para

Notice also that $P_{|\Psi_{out}\rangle} \Rightarrow \sum_i P_i^{out}$ so if we knew that the state of the particle was $|\Psi_{out}\rangle$ then we would know that it was outside the box. But the implication does not go the other way. Essentially Putnam's mistake boils down to claiming $P_R^{out} \Rightarrow P_{|\Psi_{out}\rangle}$, apparently on the mistaken assumption that we can equate $P_{|\Psi_{out}\rangle} = P_{(\sum_i P_i^{out})|\Psi\rangle}$ with $\sum_i P_i^{out}$.

new para

But at this point we can rescue Putnam's argument provided we employ the quantum-logical entailment

$$P_{|\Psi\rangle} \Rightarrow \left(P_{(\sum_i P_i^{out})|\Psi\rangle} \longleftrightarrow \sum_i P_i^{out} \right) \quad \text{— (14)}$$

which is a simple generalization of the ~~simple~~ entailment (6) employed in the Friedman–Putnam paper.

(14) says then, given that we know the

QM state is $|\psi\rangle$ then we can infer the material equivalence of the two compatible propositions associated with $P_{(\sum P_i^{out})|\psi\rangle}$ and $\sum_i P_i^{out}$. Here these propositions are compatible material equivalence is just the familiar classical Boolean notion, but the entailment is of course only valid quantum-logically (remember that $P_{|\psi\rangle}$ does not lie in the same maximal Boolean subalgebra as $P_{(\sum P_i^{out})|\psi\rangle}$ and $\sum_i P_i^{out}$). Thus knowing P_B does allow us to know $P_{(\sum P_i^{out})|\psi\rangle}$ as Putnam claims.

new para

There is certainly no hint in Putnam (1981) that he sees the need for quantum logic in arriving at his conclusion about knowing simultaneous values for incompatible observables. I submit that without this move the argument of the 1981 paper cannot be sustained.

employing

new para

So to conclude, the motivation of the move to a PBA version of quantum logic in the Friedman-Putnam paper cannot be independently established as Putnam seems to believe, but only on the basis of presupposing such a quantum logic. But there is nothing viciously circular here, indeed a pleasing consistency in the whole treatment of the two-slit problem is manifested by our discussion.

Footnotes

See back of pages 13 and 15

1.

2.

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